9.3 GEOMETRIC SEQUENCES + SERIES

A GEOMETRIC SEQUENCE IS A SEQUENCE IN WHICH EACH TERM IS THE PREVIOUS TERM MULTIPLIED BY A FIXED CONSTANT (CALLED THE COMMON RATIO r)

eg. $2 \xrightarrow{*3} 6 \xrightarrow{*3} 18 \xrightarrow{*3} 54 \xrightarrow{*3} 162 \xrightarrow{*3} 486$ GEOMETRIC

$$\frac{6}{2} = 3 = \frac{18}{6} = \frac{54}{18} = \frac{162}{54} = \frac{486}{162}$$

eg. $e^2 \xrightarrow{*e^3} e^5 \xrightarrow{*e^3} e^8 \xrightarrow{*e^3} e^{11} \xrightarrow{*e^3} e^{14} \xrightarrow{*e^3} e^{17}$ GEOMETRIC

$$\frac{e^5}{e^2} = e^3$$

eg. $64 \xrightarrow{*\frac{3}{2}} 96 \xrightarrow{*\frac{3}{2}} 144 \xrightarrow{*\frac{3}{2}} 216 \xrightarrow{*\frac{3}{2}} 324 \xrightarrow{*\frac{3}{2}} 486$ GEOMETRIC

$$\frac{\cancel{96}}{\cancel{64}} = \frac{48}{32} = \frac{24}{16} = \frac{3}{\cancel{2}} * \overset{48}{\cancel{96}} = 144$$

$$\frac{3}{\cancel{2}} * \overset{72}{\cancel{144}} = 216$$

GENERAL FORMULA FOR GEOMETRIC SEQUENCE WITH FIRST TERM a_1 AND COMMON RATIO r

$$a_1 = a_1 \qquad = a_1 r^0$$
$$a_2 = a_1 r \qquad = a_1 r^1$$
$$a_3 = a_2 r = (a_1 r) r = a_1 r^2$$
$$a_4 = a_3 r = (a_1 r^2) r = a_1 r^3$$
$$a_n \qquad = a_1 r^{n-1}$$

ARITHMETIC

$$a_n = \underbrace{a_1 +}_{\text{PLUS A FIXED CONSTANT}} \underbrace{(n-1)d}_{\text{REPEATED ADDITION OF } d}$$

GEOMETRIC

$$a_n = \underbrace{a_1}_{\text{TIMES A FIXED CONSTANT}} \underbrace{r^{n-1}}_{\text{REPEATED MULTIPLIED BY } r}$$

RECONSIDER 64, 96, 144, 216, 324, 486

$r = \frac{3}{2}$

$a_n = a_1 r^{n-1}$

$a_n = 64\left(\frac{3}{2}\right)^{n-1}$

SANITY CHECK: $a_5 = 64\left(\frac{3}{2}\right)^{5-1}$

$= 324$ ✓

FIND THE 8^{TH} TERM OF THE GEOMETRIC SEQUENCE

WITH 3^{RD} TERM $= 7$ AND 6^{TH} TERM $= -56$

$$a_8 = a_1 r^{8-1} = a_1 r^7 = \frac{7}{4}(-2)^7 = 7(-2)^5 = -7 \cdot 32 = -224$$

$6(-2)^2$

$$7 = a_3 = a_1 r^{3-1} = a_1 r^2$$

$$-56 = a_6 = a_1 r^{6-1} = a_1 r^5$$

$$a_1 r^2 = 7$$

$$a_1 r^5 = -56$$

$$\frac{\cancel{a_1} r^5}{\cancel{a_1} r^2} = \frac{-56}{7}$$

$$r^3 = -8$$

$$r = -2$$

$$a_1(-2)^2 = 7$$

$$4a_1 = 7$$

$$a_1 = \frac{7}{4}$$

GEOMETRIC SERIES

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

$$S_n = a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-1}$$

$$rS_n = r(a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-1})$$

$$rS_n = \cancel{a_1 r} + \cancel{a_1 r^2} + \cdots + \cancel{a_1 r^{n-1}} + a_1 r^n$$

$(-)$ $$S_n = a_1 + \cancel{a_1 r} + \cancel{a_1 r^2} + \cdots + \cancel{a_1 r^{n-1}}$$

$$rS_n - S_n = a_1 r^n - a_1$$

$$(r-1)S_n = a_1(r^n - 1)$$

$$S_n = \frac{a_1(r^n-1)}{r-1} \quad \text{OR} \quad \frac{a_1(1-r^n)}{1-r}$$

TYPICALLY USED IF $r > 1$ IF $r < 1$

eg. $64 + 96 + 144 + 216 + 324 + 486 \qquad r = \frac{3}{2}$

$$S_6 = \frac{64\left(\left(\frac{3}{2}\right)^6 - 1\right)}{\left(\frac{3}{2} - 1\right)} = \frac{64\left(\left(\frac{3}{2}\right)^6 - 1\right)}{\frac{1}{2}} \frac{2}{2} = 128\left(\left(\frac{3}{2}\right)^6 - 1\right)$$

WRITE IN SIGMA NOTATION

$$\frac{7}{81} + \frac{4}{54} + \frac{1}{36} - \frac{2}{24} - \frac{5}{16} = \sum_{n=1}^{5} \frac{7+(-3)(n-1)}{81\left(\frac{2}{3}\right)^{n-1}}$$

$$\frac{7}{81} + \frac{4}{54} + \frac{1}{36} + \frac{-2}{24} + \frac{-5}{16}$$

← NUMERATORS FORM ARITHMETIC SEQUENCE

$d = -3$

DENOMINATORS FORM GEOMETRIC SEQUENCE

$r = \frac{2}{3}$

$$\frac{54}{81} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{2}{\cancel{3}} * \overset{18}{\cancel{54}} = 36$$

$$\frac{2}{\cancel{3}} * \overset{12}{\cancel{36}} = 24$$

$$\frac{2}{\cancel{3}} * \overset{8}{\cancel{24}} = 16$$

$$\sum_{n=1}^{5} \frac{7-3(n-1)}{81\left(\frac{2}{3}\right)^{n-1}}$$

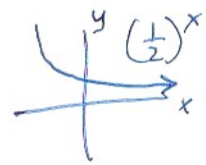
CONSIDER $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \frac{1}{48} + \cdots$ GEOMETRIC SERIES

$r = \frac{1}{2}$

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{\frac{1}{3}\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$$

AS $n \to \infty$

$\left(\frac{1}{2}\right)^n \to 0$



$$S_\infty = S = \frac{\frac{1}{3}(1-0)}{1-\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$